ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 3

DEADLINE: FRIDAY, NOVEMBER 3RD

Problem 1. The E_∞ term of the Serre spectral sequence will not determine the cohomology of the total space uniquely in general, because of extension problems. Give an example of two fibre sequences $F \to Y \to X$ with $F = \mathbb{RP}^\infty$ and $X = \mathbb{CP}^\infty$, such that the E_r pages of both Serre spectral sequences are isomorphic for all r, but $H^\bullet(E, \mathbb{Z}) \neq H^\bullet(E', \mathbb{Z})$.

Hint: First show that there are exactly two homotopy classes of maps $\mathbb{C}P^\infty \to K(Z/2, 2)$ and consider their homotopy fibers.

Problem 2. Use the Serre spectral sequence to compute $H^*(F, \mathbb{Z})$ for F the homotopy fiber of a map $S^k \to S^k$ of degree n for $k, n > 1$, and show that the cup product structure in $H^*(F, \mathbb{Z})$ is trivial.